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Magnetic-stress-assisted damping of magnetization precession in multilayered metallic films

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ABSTRACT

Micromagnetic dynamics of spin relaxation in multilayered metallic films of stacked microelectronic devices is modeled by a modified Landau–Lifshitz–Gilbert equation with a newly introduced form of damping torque owing its origin to coupling between precessing magnetization-vector and stress-tensor of combined intrinsic and extrinsic magnetic anisotropy. Based on the magnetization energy loss equation, the exponential relaxation time as a function of precession frequency and angle of applied rf-field is obtained, depending upon two parameters of intrinsic and extrinsic damping torques acting on precessing magnetization. It is shown that theoretically obtained from the Gabor uncertainty relation the FMR linewidth, originating from the above magnetic-stress-assisted damping of magnetization precession, provides proper account for the empirical non-linear linewidth-vs-frequency curves deduced from recent in-plane FMR measurements on multilayered ultrathin films of ferromagnetic metals.

1. Introduction

An understanding of relaxation processes in ultrathin films of ferromagnetic metals is crucial to the design and construction of microelectronic devices [1,2], like magnetic random access memory (MRAM) and spatial light modulators (SLM). The main source of information about relaxation processes has been and still is the ferromagnetic resonance (FMR) measurements aimed at revealing the frequency dependence of the full-width-at-half-maximum in FMR spectral line [3]. Traditionally, the results of these experiments are treated within the framework of the phenomenological Landau–Lifshitz–Gilbert model (e.g. [4,5]), describing the FMR response in terms of uniform precession of magnetization $\mathbf{M}(t)$ with the preserved in time magnitude $|\mathbf{M}(t)| = M_s$ where M_s is the saturation magnetization. The dynamical equation governing precessional motions of $\mathbf{M}(t)$ can be conveniently written in the following general form:

$$\dot{\mathbf{M}}(t) = -\gamma\mu_0 \mathbf{T}(t) - \mathbf{R}(t), \quad (1)$$

where γ is the electronic gyromagnetic ratio and μ_0 is the magnetic permeability of free space; SI units are used throughout this paper. The vector-function, $\mathbf{T}(t) = [\mathbf{M}(t) \times \mathbf{H}]$, represents the magnetic torque that drives the free Larmor precession of $\mathbf{M}(t)$ about the axis of the dc magnetic field, $\mathbf{H} = \text{constant}$, in the process of which the Zeeman magnetization energy, $W_m(t) = -\mu_0 \mathbf{H} \cdot \mathbf{M}(t)$, is conserved: $\dot{W}_m(t) = 0$. Central to understanding the relaxation process is the damping torque $\mathbf{R}(t)$ carrying information about peculiarities of aligning $\mathbf{M}$ with $\mathbf{H}$ and defining the rate of magnetization energy loss

$$\dot{W}_m(t) = -\mu_0 \mathbf{H} \cdot \dot{\mathbf{M}}(t) = \mu_0 \mathbf{H} \cdot \mathbf{R}(t). \quad (2)$$

Of particular interest to the subject of our present study is the original Landau–Lifshitz (LL) form of this function $\mathbf{R}(t) = \lambda_{in}[\mathbf{M}(t) \times [\mathbf{M}(t) \times \mathbf{H}]]$ where the material-dependent parameter λ_{in} is thought of as measuring the strength of the intrinsic anisotropy on the relaxation dynamics of magnetization precession which is constrained by the conditions $\mathbf{M}(t) \cdot \dot{\mathbf{M}}(t) = 0$ and $\mathbf{M}(t) \cdot \mathbf{R}(t) = 0$. In this paper we discuss an alternative (to original geometrical) treatment of $\mathbf{R}(t)$ according to which the origin of the damping torque responsible for spin relaxation in multilayered metallic films is attributed to the coupling between the uniformly precessing magnetization-vector and the stress-tensor of combined intrinsic and extrinsic magnetic anisotropy.

2. Stress-tensor representation of micromagnetic damping torque

The equilibrium magnetic anisotropy exhibited in the easy and hard axes of magnetization direction is a hallmark of ultrathin films of ferromagnetic metals. Viewing this property from the perspective of the macroscopic electrodynamics of magnetic continuous media [6], it seems quite natural to invoke the stress-tensor description of magnetic anisotropy, namely, in terms of

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symmetric tensors of magnetic-field-dependent stresses composed of $M\otimes H$ and $H\otimes H$ components where by $\mathbf{H}$ is understood total (applied and internal effective) field. In so doing we adopt the following definition of stress-tensor of intrinsic anisotropy $\sigma_{lk}^{\rm in}$ (generic to both monolithic and multilayered ferromagnetic films) and stress-tensor of extrinsic anisotropy $\sigma_{ik}^{\rm ex}$ (arising from impurities and imperfections of the film crystalline lattice)

$$\sigma_{lk}^{\rm in} = \frac{\mu}{2}[(M_nH_n)\delta_{kl} - (M_lH_k + M_kH_l)], \tag{3}$$

$$\sigma_{ik}^{\rm ex} = \frac{\mu}{2}[H^2\delta_{kl} - (H_lH_k + H_kH_l)], \tag{4}$$

where δ_{ik} is the Kronecker symbol and μ stands for the effective magnetic permeability which is derived from the magnetization curve according to the rule [7]: $\mu = \Delta B/\Delta H$. It is worth noting that the above stress-tensor description of intrinsic and extrinsic magnetic anisotropy is consistent with the definition of the energy density of magnetic field stored in a ferromagnetic film (which can be found, for instance, in [8])

$$u = \tfrac{1}{2}\mathbf{B}\cdot\mathbf{H}, \quad \mathbf{B} = \mu(\mathbf{H}+\mathbf{M}) \tag{5}$$

in the sense that the relation between the stress-tensor of combined intrinsic and extrinsic magnetic anisotropy, $\sigma_{lk} = \sigma_{lk}^{\rm in} + \sigma_{lk}^{\rm ex}$, and the energy density u is described by

$$u = {\rm tr}[\sigma_{lk}] = \sigma_{ll} = \frac{\mu}{2}(MH + H^2), \tag{6}$$

where ${\rm tr}[\sigma_{lk}]$ stands for the trace of tensor σ_{lk}. In what follows we focus on the effect of above magnetic stresses on the precessing magnetization vector whose mathematical treatment is substantially relied on the symmetric tensor

$$\gamma_{ik} = [M_s^2\delta_{ik} - M_iM_k], \quad \gamma_{ik} = \gamma_{ki}, \tag{7}$$

having, in appearance, some features in common with that for isotropic magneto-striction stresses [9]. It can be verified by direct calculation that the stress-tensor representation of the intrinsic relaxation function is identical to the LL relaxation function

$$R_i^{\rm in} = \frac{2\lambda_{\rm in}}{\mu M_s^2}\gamma_{ik}M_l\sigma_{kl}^{\rm in} = \lambda_{\rm in}[M_i(M_kH_k) - H_iM_s^2],$$
$$\mathbf{R}_{\rm in} = \lambda_{\rm in}[\mathbf{M}(t)\times[\mathbf{M}(t)\times\mathbf{H}]]. \tag{8}$$

In choosing the above form of the tensor γ_{ik} we were guided by previous investigations [10] of the damping terms in ferro-nematic liquid crystals dealing with the tensor constructions of a similar form. For the extrinsic relaxation function, owing its origin to the coupling of M_l with $\sigma_{lk}^{\rm ex}$, we use the following stress-tensor representation

$$R_i^{\rm ex} = -\frac{\lambda_{\rm ex}}{\mu\Omega}\gamma_{ik}M_l\sigma_{lk}^{\rm ex}$$
$$= -\lambda_{\rm ex}\frac{(M_nH_n)}{\Omega}[M_i(M_kH_k) - H_iM_s^2]. \tag{9}$$

The minus sign means that the extrinsic damping torque counteracts the damping torque originating from the intrinsic stresses. The vector form of extrinsic relaxation function (9) reads

$$\mathbf{R}_{\rm ex}(t) = -\lambda_{\rm ex}\frac{(\mathbf{M}(t)\cdot\mathbf{H})}{\Omega}[\mathbf{M}(t)\times[\mathbf{M}(t)\times\mathbf{H}]]. \tag{10}$$

As is shown below, the parameter-free frequency of the transient magnetization configuration

$$\Omega = \frac{\omega}{1-(\omega_M/\omega)^{1/2}}, \quad \omega_M = \gamma\mu_0M_s, \;\; \omega = 2\pi f, \tag{11}$$

provides correct physical dimension of the extrinsic damping torque and proper account for the empirical dependence of the FMR linewidth ΔH upon the resonance frequency f. Making use of argument of physical dimension it is easy to show that the material-dependent parameters $\lambda_{\rm in} > 0$ and $\lambda_{\rm ex} > 0$ (measuring the strength effect of intrinsic and extrinsic stresses on the relaxation process in multilayered film) can be represented in terms of dimensionless damping constants α and β (whose magnitudes are deduced from the empirical frequency dependence of FMR linewidth) as follows:

$$\lambda_{\rm in} = \alpha\frac{\gamma\mu_0}{M_s}, \quad \lambda_{\rm ex} = \beta\left(\frac{\gamma\mu_0}{M_s}\right)^2. \tag{12}$$

The net outcome of the above outlined procedure of computing the combined (intrinsic plus extrinsic) damping torque

$$\mathbf{R} = \mathbf{R}_{\rm in} + \mathbf{R}_{\rm ex}$$
$$= \left[\lambda_{\rm in} - \lambda_{\rm ex}\frac{(\mathbf{M}(t)\cdot\mathbf{H})}{\Omega}\right][\mathbf{M}(t)\times[\mathbf{M}(t)\times\mathbf{H}]], \tag{13}$$

entering the basic equation of micromagnetic dynamics (1) is the following Modified Landau–Lifshitz (MLL) equation

$$\dot{\mathbf{M}} = -\gamma\mu_0[\mathbf{M}\times\mathbf{H}]$$
$$-\left[\alpha\frac{\gamma\mu_0}{M_s} - \beta\left(\frac{\gamma\mu_0}{M_s}\right)^2\frac{(\mathbf{M}\cdot\mathbf{H})}{\Omega}\right][\mathbf{M}\times[\mathbf{M}\times\mathbf{H}]], \tag{14}$$

which obeys all constraints of the canonical LL equation. One sees that unlike the linear-in-magnetic-field intrinsic damping torque, the extrinsic damping torque is described by quadratic-in-magnetic-field relaxation function. At this point it seems noteworthy that the need in allowing for the quadratic-in-H damping terms has been discussed long ago [11]. The above scheme can be regarded, therefore, as a development of this line of theoretical investigations. In terms of the unit vector of magnetization $\mathbf{m}(t) = \mathbf{M}(t)/M_s$ and Larmor frequency $\boldsymbol{\omega} = \gamma\mu_0\mathbf{H}$ the last equation can be converted to (see [12] for comparison)

$$\dot{\mathbf{m}} = [\boldsymbol{\omega}\times\mathbf{m}] - \left[\alpha - \beta\frac{(\mathbf{m}\cdot\boldsymbol{\omega})}{\Omega}\right][\mathbf{m}\times[\mathbf{m}\times\boldsymbol{\omega}]]. \tag{15}$$

It can be seen that the obtained MLL equations (14) and (15) are reduced to the standard LL equation when the effect of extrinsic stresses is ignored (i.e. $\beta = 0$).

3. Relaxation time and FMR linewidth

The relaxation time is amongst the primary targets of current FMR experiments. In this section, we present variational method of analytic computation of the FMR linewidth which is quite different from the well-known susceptibility solution of LL equation (e.g. [5]). At the base of the variation method under consideration lies the equation of the magnetization energy loss from which the exponential relaxation time τ as a function of FMR frequency $f = \omega/(2\pi)$ is derived. The FMR linewidth, $\Delta\omega = \gamma\mu_0\Delta H$, is computed from the well-known Gabor uncertainty relation (e.g. [13], Section 11.2)

$$\Delta\omega\,\tau = 1 \tag{16}$$

between the full-width-at-half-maximum in the resonance-shaped spectral line $\Delta\omega$ and lifetime τ of resonance excitation.

3.1. FMR linewidth caused by intrinsic damping torque

For the former we consider relaxation process brought about by intrinsic damping torque $R_i^{\rm in}(t)$ given by Eq. (8). Our approach is based on the observation that the equation of magnetization energy loss in the process of a uniform precession of magnetization in a dc magnetic field

$$\frac{dW_m}{dt} = -\mu_0HM_s\frac{d(\cos\theta(t))}{dt} = \mu_0H_iR_i^{\rm in}(t), \tag{17}$$

is reduced to the equation for the cosine function $u(t) = \cos\theta(t)$ of angle $\theta(t)$ between $\mathbf{M}(t)$ and $\mathbf{H}$, namely

$$\frac{du(t)}{dt} = -\alpha\omega[u^2(t)-1], \quad \omega = \gamma\mu_0 H. \tag{18}$$

The right hand side of (18) suggests that there are two equilibrium configurations, namely, with $u_0(0)=1$ corresponding to $\mathbf{M}\uparrow\uparrow\mathbf{H}$ and $u_0(\pi)=-1$ corresponding to $\mathbf{M}\uparrow\downarrow\mathbf{H}$. The stability of these configurations can be assessed by the standard procedure of introducing small-amplitude deviations $\delta u(t)$ from the equilibrium values $u_0=\pm 1$. On substituting $u(t)=u_0+\delta u(t)$, into (18) with $u_0=1$ and retaining first order terms in $\delta u(t)$ we obtain equations describing exponential relaxation of magnetization to the state of stable magnetic equilibrium:

$$\frac{d\delta u(t)}{dt} = -(2\alpha\omega)\delta u \to \delta u(t) = \delta u(0)e^{-t/\tau}, \tag{19}$$

$$\tau^{-1} = 2\alpha\omega, \quad \omega = 2\pi f. \tag{20}$$

The second stationary state, with $u_0=-1$, is unstable, since in this case the resultant linearized equation, $\delta\dot{u}=(2\alpha\omega)\delta u$, having the solution, $\delta u(t)=\delta u(0)e^{t/\tau}$, describes a non-physical behavior of δu as the time is increased. Inserting (20) in (16), we arrive at the basic prediction of the standard micromagnetic model

$$\Delta H(f) = Af, \quad A = \frac{4\pi\alpha}{\gamma\mu_0}. \tag{21}$$

This last equation provides a basis for discussion of empirical linewidth-frequency dependence $\Delta H_{exp}=\Delta H_0+\Delta H_{exp}(f)$ with $\Delta H_{exp}(f)=A_{exp}f$. Central to such a discussion is the identification of theoretical and experimental linewidths, $\Delta H(f)=\Delta H_{exp}(f)$, from which the magnitude of α is deduced and applied to (20) for obtaining numerical estimates of the relaxation time τ.

3.2. FMR linewidth caused by both intrinsic and extrinsic damping torques

In this case the starting point is the equation of magnetization energy loss

$$\frac{dW_m}{dt} = -\mu_0 H M_s \frac{d(\cos\theta(t))}{dt} = \mu_0 H_i R_i(t), \tag{22}$$

where $R_i(t)=R_i^{\rm in}(t)+R_i^{\rm ex}(t)$ is the combined relaxation function of intrinsic and extrinsic damping, Eq. (9). Taking again $u(t)=\cos\theta(t)$ as a basic dynamical variable of relaxation process, one finds that the last equation is reduced, after some algebra, to

$$\frac{du(t)}{dt} = -\left[\alpha\omega-\beta\frac{\omega^2}{\Omega}u(t)\right](u^2(t)-1). \tag{23}$$

The right hand side of this equation indicates that there are three stationary state characterized by

$$u_0(\theta=0,\pi)=\pm 1, \quad u_0(\theta=\theta_M)=\frac{\alpha}{\beta}\frac{\Omega}{\omega}. \tag{24}$$

Applying to (23) the standard linearization procedure $u(t)=u_0+\delta u(t)$, it is easy to see that resultant equation is equivalent to the equation of exponential relaxation, $\delta\dot{u}=-\tau^{-1}\delta u$, if and only if the parameter

$$\tau^{-1} = \left[2\left(\alpha\omega-\beta\frac{\omega^2}{\Omega}u_0\right)u_0+\beta\frac{\omega^2}{\Omega}(1-u_0^2)\right] \tag{25}$$

is a positive constant. It is easy to see that this is the case for $u_0(\theta=0)=1$ and $u_0(\theta=\theta_M)$ given by rightmost of Eq. (24). This latter u_0 corresponds to a quasi-stationary transient configuration of precessing magnetization owing its existence to the coupling of magnetization with extrinsic stresses of magnetic anisotropy. The state with $u_0(\pi)=-1$, is unstable. For the total relaxation time $\tau^{-1}=\tau^{-1}(\theta=0)+\tau^{-1}(\theta=\theta_M)$ and the FMR linewidth (following from Gabor uncertainty relation $\Delta H=[\gamma\mu_0\tau]^{-1}$) we obtain

$$\tau^{-1} = 2\alpha\omega-\beta\frac{\omega^2}{\Omega}(1+\cos^2\theta_M), \tag{26}$$

$$\Delta H = \frac{2\omega}{\gamma\mu_0}\left(\alpha-\frac{\beta\omega}{2\Omega}[1+\cos^2\theta_M]\right) = \frac{4\pi f}{\gamma\mu_0}\left[\alpha-\frac{\beta}{2}\left(1-\left(\frac{\gamma\mu_0 M_s}{2\pi f}\right)^{1/2}\right)(1+\cos^2\theta_M)\right]. \tag{27}$$

It is easy to see that in the limit of $\beta=0$, the last two equations, representing the basic predictions of the developed theory, are reduced to those for the standard LLG model in which the effect of extrinsic stresses on magnetization precession is ignored.

4. Discussion and summary

In approaching the interpretation of FMR measurements in terms of presented theory, in the remainder of this work, we focus on a case of in-plane configuration ($\mathbf{M}\uparrow\uparrow\mathbf{H}$) which is of particular interest in connection with the recent discovery of non-linear frequency dependence of FMR linewidth [3,15–17]. In this case, the last equation for the FMR linewidth takes the form

$$\Delta H = \frac{4\pi f}{\gamma\mu_0}[\alpha-\beta(1-(\gamma\mu_0 M_s/2\pi f)^{1/2})]. \tag{28}$$

In below presented numerical simulations on extracting damping constants α and β from data of FMR measurements we use $\gamma=1.76\times 10^{11}\ \mathrm{T^{-1}\ s^{-1}}$ and $M_s=8400$ kA/m. To illuminate the difference between predictions of the standard and modified LL models, in Fig. 1 we plot τ and ΔH as functions of the FMR frequency f computed with the pointed out parameters of α and β. In computation based on the standard micromagnetic model, Eq. (21), we have used the same value of α as in work [14] reporting the FMR measurements on ultrathin films of Permalloy. The presented in Fig. 1 values of α and β have been deduced from fitting, Eq. (28), of the non-linear frequency dependence of FMR linewidth discovered in the FMR measurements [15]. The result of this fit, demonstrating potential of developed theory, is shown in upper panel of Fig. 2. In lower panel of Fig. 2, we plot our fit of data on the FMR linewidth measurements [16] on multilayered thin films of Fe/V. It is seen that the micromagnetic model relying on MLL-equation properly regain a non-linear character of empirical linewidth-frequency dependence in low-frequency region. Fig. 3

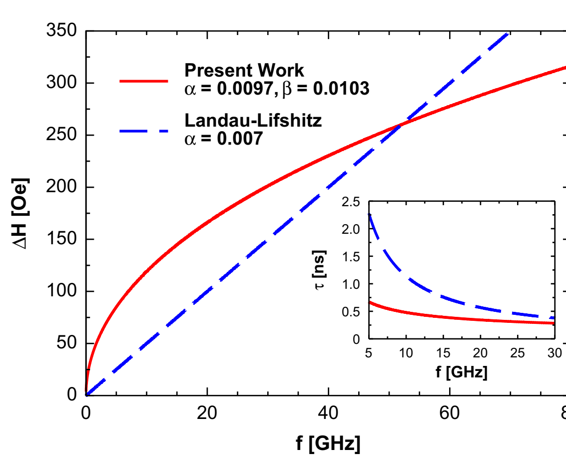

Fig. 1. Plot of theoretical FMR linewidth ΔH(Oe) as a function of the FMR frequency f(GHz), computed, with pointed out parameters, on the basis of standard LL-equation and modified LL-equation of present work. The presented in the insertion plot of the frequency dependence of exponential relaxation time τ shows that relaxation process governed by MLL equation falls in the picosecond regime.

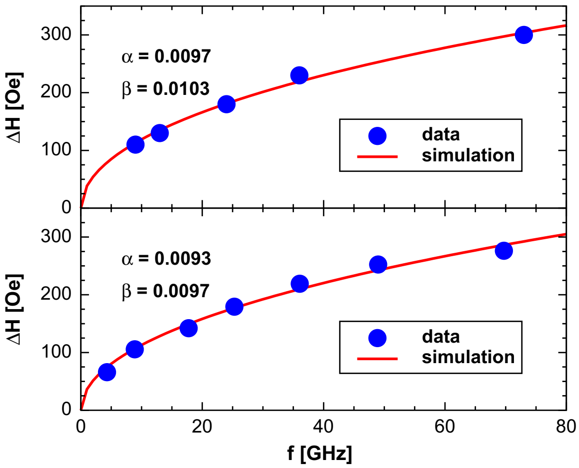

Fig. 2. The solid line is a plot of FMR linewidth ΔH(Oe) as a function of frequency f(GHz) computed from two-parametric equation (28). The pointed out damping parameters α and β provide a best fit of experimental data on FMR linewidth vs. frequency (dots) obtained from in-plane FMR measurements [15] on multilayered metallic nanostructures Pd/Fe/GaAs (upper panel) and data of in-plane FMR measurements on thin films Fe/V (lower panel) reported in [16]. The figures clearly exhibit a non-linear character of the FMR linewidth–frequency curves which, in the considered micromagnetic model relaying on MLL equation, is attributed to combined effect of intrinsic and extrinsic damping terms on precessing magnetization vector.

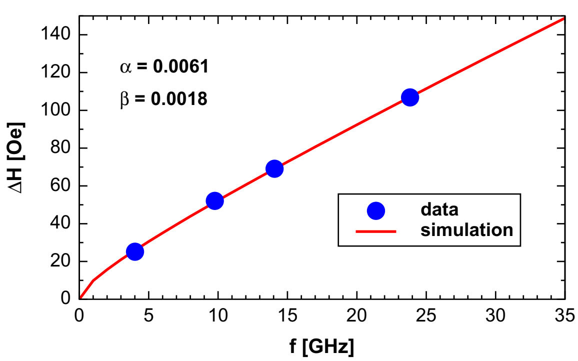

Fig. 3. Plot of theoretical FMR linewidth ΔH(Oe) as a function of frequency f(GHz), Eq. (28), computed with pointed out damping constants α and β in juxtaposition with data on in-plane FMR measurements [17] on polycrystalline Co films grown on the GaAS substrate.

shows theoretical fit of in-plane FMR measurements [17] on ultrathin polycrystalline Co film grown on GaAS substrate. The predictions of Eq. (26) regarding angular dependence of the FMR linewidth are demonstrated in Fig. 4. This figure clearly shows that the non-linear dependence of linewidth on frequency should be profoundly manifested in case of in-plane configuration of magnetization ($\theta = 0$). In Fig. 5 we plot angular dependence of FMR linewidth computed at different frequencies. The results of our simulations presented in this latter figure are in fairly good agreement with data presented in [3].

As a summary, the considered micromagnetic mechanism of the magnetization precession damping (due to magnetization-stress coupling) presumes that the process of spin-relaxation is not accompanied by generation of spin-waves (magnons), because the magnetization **M** is regarded as a spatially-uniform vector across the multilayered film. At this point the considered regime of the magnon-free spin relaxation (in which the wave vector of spin wave $k = 0$) is quite different from spin relaxation caused by two-magnon scattering [18]. The most conspicuous feature of considered (substantially macroscopic) mechanism, responsible for the non-linear frequency dependence of FMR linewidth, is the transient

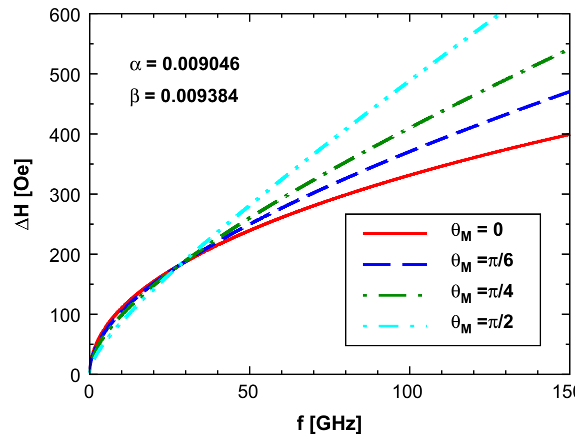

Fig. 4. The frequency dependence of FMR linewidth upon field angle computed from Eq. (26).

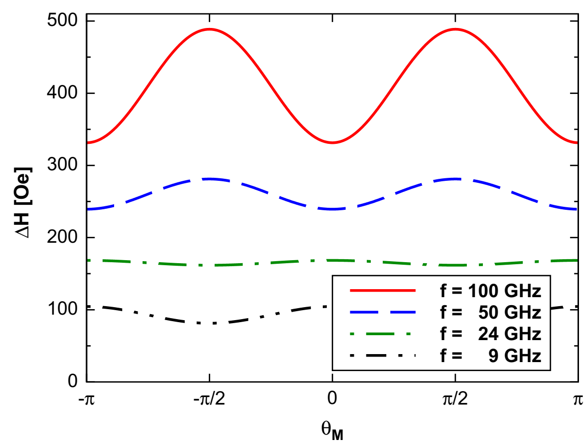

Fig. 5. The FMR linewidth versus angle between vectors of applied field and magnetization computed with the pointed out FMR frequencies, (26).

magnetization configuration owing its existence to the extrinsic stresses generic to the multilayered films. Such a configuration is absent in perfect monolithic films (without impurities and defects of crystalline lattice) whose ferromagnetic properties are dominated by intrinsic stresses of magnetic anisotropy.

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