# Micromagnetic theory of spin relaxation and ferromagnetic resonance in multilayered metallic films

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Spin relaxation in the ultrathin metallic films of stacked microelectronic devices originating from the coupling between precessing vector of magnetization and stress-tensors of intrinsic and extrinsic magnetic anisotropy is investigated on the basis of derived modified Landau-Lifshitz equation of micromagnetic dynamics. Particular attention is given to variation method of computing the time of exponential relaxation and ferromagnetic resonance linewidth relying on equation of the magnetization energy loss and Gabor uncertainty relation between the full-width-at-half-maximum in resonance-shaped line and lifetime of resonance excitation.

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## I. INTRODUCTION

An understanding of relaxation processes in ultrathin films of ferromagnetic metals is crucial to the design and construction of microelectronic devices like magnetic random access memory and spatial light modulator[1,2]. The main source of information about relaxation processes have been and still remind the ferromagnetic resonance (FMR) measurements aimed at revealing the frequency dependence of the full-width-at-half-maximum in FMR spectral line (e.g[3,4]). Traditionally, the results of these experiments are treated within the framework of phenomenological Landau-Lifshitz-Gilbert model[5–7] describing the FMR response in terms of uniform precession of magnetization $\mathbf{M}(t)$ with the preserved in time magnitude $|\mathbf{M}(t)| = M_s$ where $M_s$ is the saturation magnetization. The dynamical equation governing precessional motions of $\mathbf{M}(t)$ can be conveniently written in the following general form

$$\dot{\mathbf{M}}(t) = -\gamma\mu_0\mathbf{T}(t) - \mathbf{R}(t), \tag{1}$$

where $\gamma$ is the electronic gyromagnetic ratio and $\mu_0$ is the magnetic permeability of free space; SI units are used throughout this paper. The vector-function, $\mathbf{T}(t) = [\mathbf{M}(t) \times \mathbf{H}]$, stands for the magnetic torque that drives the free Larmor precession of $\mathbf{M}(t)$ about axis of dc magnetic field, $\mathbf{H} =$ constant, in the process of which the Zeeman magnetization energy, $W_m(t) = -\mu_0\mathbf{H} \cdot \mathbf{M}(t)$, is conserved: $\dot{W}_m(t) = 0$. Central to understanding of the magnetization energy loss which is controlled by equation

$$\dot{W}_m(t) = -\mu_0\mathbf{H} \cdot \dot{\mathbf{M}}(t) = \mu_0\mathbf{H} \cdot \mathbf{R}(t) \tag{2}$$

is the relaxation function $\mathbf{R}(t)$. In this work we use the Landau-Lifshitz (LL) form of this function

$$\mathbf{R}_{in}(t) = \lambda_{in}[\mathbf{M}(t) \times [\mathbf{M}(t) \times \mathbf{H}]], \tag{3}$$

providing geometrically transparent insight into the magnetization-vector motion in the process of aligning $\mathbf{M}$ with $\mathbf{H}$. The material-dependent parameter $\lambda_{in}$ is thought of as describing the strength effect of intrinsic anisotropy on relaxation dynamics of magnetization precession which is constrained by conditions $\mathbf{M}(t) \cdot \dot{\mathbf{M}}(t) = 0$ and $\mathbf{M}(t) \cdot \mathbf{R}(t) = 0$. Not withstanding the standard geometrical arguments, in this paper we consider an alternative micromagnetic treatment of $\mathbf{R}(t)$ according to which the origin of damping torque responsible for spin relaxation in multilayered metallic films is attributed to the coupling between the uniformly precessing magnetization-vector and the stress-tensors of intrinsic and extrinsic magnetic anisotropy.

## II. STRESS-TENSOR REPRESENTATION OF MICROMAGNETIC DAMPING TORQUE

The equilibrium magnetic anisotropy exhibited in the easy and hard axises of magnetization direction[8] is a hallmark of ultrathin films of ferromagnetic metals. Regarding this property from perspective of the macroscopic electrodynamics of magnetic continuous media[9,10], it seems quite natural to invoke the stress-tensor description of magnetic anisotropy, namely, in terms of symmetric tensors of magnetic-field-dependent stresses. In so doing we adopt the following definition of stress-tensor of intrinsic anisotropy $\sigma^{\rm in}_{lk}$ (generic to both monolithic and multilayered ferromagnetic films) and stress-tensor of extrinsic anisotropy $\sigma^{\rm ex}_{ik}$ (arising from impurities and imperfections of the film crystalline lattice)

$$\sigma^{\rm in}_{lk} = \frac{\mu}{2}\left[(M_nH_n)\delta_{kl} - (M_lH_k + M_kH_l)\right], \tag{4}$$

$$\sigma^{\rm ex}_{ik} = \frac{\mu}{2}\left[H^2\delta_{kl} - (H_lH_k + H_lH_k)\right], \tag{5}$$

where $\delta_{ik}$ is the Kronecker symbol and $\mu$ stands for the effective magnetic permeability which is derived from the magnetization curve according to the rule[11]: $\mu = \Delta B/\Delta H$. Hereafter by $\mathbf{H}$ is understood the total (applied and internal effective) field. It is worth noting that the above stress-tensor description of intrinsic and extrinsic magnetic anisotropy is consistent with definition of the energy density of magnetic field stored in a ferromagnetic film[12]

$$u = \frac{1}{2}\mathbf{B}\cdot\mathbf{H}, \quad \mathbf{B} = \mu(\mathbf{H} + \mathbf{M}) \tag{6}$$

in the sense that the interrelation between stress-tensor of combined intrinsic and extrinsic magnetic anisotropy, $\sigma_{lk} = \sigma^{in}_{lk} + \sigma^{ex}_{lk}$, and the energy density $u$ is described by

$$u = Tr[\sigma_{lk}] = \sigma_{ll} = \frac{\mu}{2}(MH + H^2), \tag{7}$$

where $Tr[\sigma_{lk}]$ stands for the trace of $\sigma_{lk}$. In what follows we focus on the effect of above magnetic stresses on precessing magnetization vector whose mathematical treatment is substantially relied on the symmetric tensor

$$\gamma_{ik} = [M_s^2\delta_{ik} - M_iM_k], \quad \gamma_{ik} = \gamma_{ki}, \tag{8}$$

having, in appearance, some features in common with that for isotropic magneto-striction stresses[13]. It can be verified by direct calculation that the stress-tensor representation of the intrinsic relaxation function

$$R^{in}_i = \frac{2\lambda_{in}}{\mu\, M_s^2}\gamma_{ik}\, M_l\, \sigma^{in}_{kl} = \lambda_{in}[M_i(M_kH_k) - H_iM_s^2] \tag{9}$$

is identical to the vector form of LL relaxation function (3). In choosing the above form of tensor $\gamma_{ik}$ we were guided by previous investigations[14] of the damping terms in ferro-nematic liquid crystals dealing with the tensor constructions of a similar form. Stress-tensor representation of the extrinsic relaxation function, owing its origin to the coupling of $M_l$ with $\sigma_{lk}^{ex}$, is given by

$$R_i^{\rm ex} = \frac{2\lambda_{\rm ex} M_s^2}{\mu\Omega}\gamma_{ik} M_l \sigma_{lk}^{ex} \tag{10}$$
$$= -\lambda_{\rm ex}\frac{(M_n H_n)}{\Omega}[H_i(M_k M_k) - M_i(M_k H_k)].$$

The vector form of this latter function reads

$$\mathbf{R}_{ex}(t) = -\lambda_{ex}\frac{(\mathbf{M}(t)\cdot\mathbf{H})}{\Omega}\,[\mathbf{M}(t)\times[\mathbf{M}(t)\times\mathbf{H}]]. \tag{11}$$

As is shown below, the parameter-free frequency of the transition magnetization configuration

$$\Omega = \frac{\omega}{1-(\omega_M/\omega)^{1/2}},\ \omega_M = \gamma\mu_0 M_s,\ \omega = 2\pi f, \tag{12}$$

provides correct physical dimension of the extrinsic damping torque and proper account for the empirical dependence of the FMR linewidth $\Delta H$ upon the resonance frequency $f$. Making use of argument of physical dimension it easy to show that the material-dependent parameters $\lambda_{in} > 0$ and $\lambda_{ex} > 0$ (measuring, as was noted, the strength effect of intrinsic and extrinsic anisotropies on the relaxation process in multilayered film) can be represented in terms of dimensionless damping constants $\alpha$ and $\beta$ (whose magnitudes are deduced from the empirical frequency dependence of FMR linewidth) as follows

$$\lambda_{in} = \alpha\frac{\gamma\mu_0}{M_s},\quad \lambda_{ex} = \beta\left(\frac{\gamma\mu_0}{M_s}\right)^2. \tag{13}$$

The net outcome of above outlined procedure of computing the combined damping torque $\mathbf{R} = \mathbf{R}_{in} + \mathbf{R}_{ex}$, originating from coupling of magnetization with both intrinsic and extrinsic stresses, is the following Modified Landau-Lifshitz (MLL) equation

$$\dot{\mathbf{M}} = -\gamma\mu_0[\mathbf{M}\times\mathbf{H}] - \alpha\frac{\gamma\mu_0}{M_s}[\mathbf{M}\times[\mathbf{M}\times\mathbf{H}]] + \beta\left(\frac{\gamma\mu_0}{M_s}\right)^2\frac{(\mathbf{M}\cdot\mathbf{H})}{\Omega}[\mathbf{M}\times[\mathbf{M}\times\mathbf{H}]]$$
$$= -\gamma\mu_0[\mathbf{M}\times\mathbf{H}] - \frac{\gamma\mu_0}{M_s}\left[\alpha - \beta\left(\frac{\gamma\mu_0}{M_s}\right)\frac{(\mathbf{M}\cdot\mathbf{H})}{\Omega}\right][\mathbf{M}\times[\mathbf{M}\times\mathbf{H}]], \tag{14}$$

which obeys all constrains of the canonical LL equation. One sees that unlike the linear-in-magnetic-field intrinsic damping torque, the extrinsic damping torque is described by quadratic-in-magnetic-field relaxation function. At this point it seems noteworthy that the need in allowing for damping terms quadratic in $H$ has been discussed long ago in[15]. The above expounded scheme can be regarded, therefore, as a development of this line of theoretical investigations. In terms of the unit vector of magnetization $\mathbf{m} = \mathbf{M}/M_s$ and Larmor frequency $\boldsymbol{\omega} = \gamma\mu_0\mathbf{H}$ the last equation can be converted to (see for comparison[16])

$$\dot{\mathbf{m}} = [\boldsymbol{\omega} \times \mathbf{m}] - \left[\alpha - \beta\,\frac{(\mathbf{m}\cdot\boldsymbol{\omega})}{\Omega}\right][\mathbf{m} \times [\mathbf{m} \times \boldsymbol{\omega}]]. \tag{15}$$

It can be seen that obtained MLL equations (14) and (15) are reduced to the standard LL equation when the effect of extrinsic stresses is ignored ($\beta = 0$).

## III. VARIATION METHOD OF COMPUTING RELAXATION TIME AND FMR LINEWIDTH

The relaxation time is amongst the primary targets of current FMR experiments. In this section, we present variation method of computing relaxation time $\tau$ as a function of FMR frequency $f = \omega/(2\pi)$. Unlike the well-known for a long time susceptibility solution of LL equation[7], the variational approach under consideration rests on equation of the magnetization energy loss from which the time of exponential relaxation of magnetization vector is derived. The FMR linewidth, $\Delta\omega = \gamma\mu_0\Delta H$, is computed from the well-known Gabor uncertainty relation (e.g.[17], Sec.11.2)

$$\Delta\omega\,\tau = 1 \tag{16}$$

between the full-width-at-half-maximum in the resonance-shaped spectral line $\Delta\omega$ and lifetime $\tau$ of resonance excitation.

### A. FMR linewidth caused by intrinsic damping torque

For the former we consider relaxation process brought about by intrinsic damping torque. Our approach is based on observation that the equation of magnetization energy loss in the process of a uniform precession of magnetization in dc magnetic field

$$\frac{dW_m}{dt} = -\mu_0 H M_s \frac{d(\cos\theta(t))}{dt} = \mu_0 H_i R_i^{in}(t), \tag{17}$$

$$R_i^{in} = \frac{2\lambda_{in}}{\mu\, M_s^2}\gamma_{ik}\, M_l\, \sigma_{kl}^{in}, \tag{18}$$

is reduced to equation for the cosine function $u(t) = \cos\theta$ of angle $\theta$ between $\mathbf{M}(t)$ and $\mathbf{H}$, namely

$$\frac{du(t)}{dt} = -\alpha\omega[u^2(t) - 1], \quad \omega = \gamma\mu_0 H. \tag{19}$$

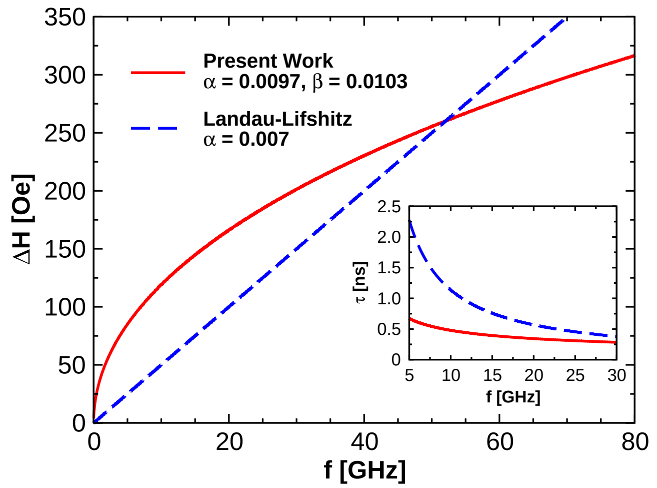


FIG. 1. The FMR linewidth $\Delta H$ and relaxation time $\tau$ as functions of the FMR frequency $f$, computed on the basis of standard and modified in present work Landau-Lifshitz equation with pointed out damping constants the choice of which is discussed in the text.

The right hand side of (19) suggests that there are two equilibrium configurations, namely, with $u(0) = 1$ corresponding to $\mathbf{M} \uparrow\uparrow \mathbf{H}$ and $u(0) = -1$ to $\mathbf{M} \uparrow\downarrow \mathbf{H}$. The stability of these configurations can be assessed by the standard procedure of introducing the small-amplitude deviations $\delta u(t)$ of $u$ from the above equilibrium values $u(0) = u_0 = \pm 1$. On substituting $u(t) = u_0 + \delta u(t)$, in (19) $u_0 = 1$ and retaining terms of the first order in $\delta u(t)$ we obtain equation

$$\frac{d\delta u(t)}{dt} = -(2\alpha\omega)\delta u \;\rightarrow\; \delta u(t) = \delta u(0)\mathrm{e}^{-t/\tau}, \tag{20}$$

$$\tau^{-1} = 2\alpha\omega, \quad \omega = \gamma\mu_0 H. \tag{21}$$

describing exponential relaxation of magnetization to the state of stable magnetic equilibrium. On inserting (21) in (16), we arrive at the basic prediction of the standard micromagnetic model

$$\Delta H(f) = A\, f, \quad A = \frac{4\pi\alpha}{\gamma\mu_0}. \tag{22}$$

This last equation provides a basis for discussion of empirically established linewidth-frequency dependence $\Delta H_{exp} = \Delta H_0 + \Delta H_{exp}(f)$ with $\Delta H_{exp}(f) = A_{exp}\, f$. Central to such a discussion is the identification of theoretical and experimental linewidths, $\Delta H(f) = \Delta H_{exp}(f)$, from which the magnitude of $\alpha$ is deduced and applied to (21) for obtaining numerical estimates of relaxation time $\tau$.

### B. FMR linewidth caused by both intrinsic and extrinsic damping torques

In this case the starting point is the equation of magnetization energy loss with the combined relaxation function

$$\frac{dW_m}{dt} = -\mu_0 H M_s \frac{d(\cos\theta(t))}{dt} = \mu_0 H_i R_i(t), \tag{23}$$

$$R_i = \frac{2\lambda_{in}}{\mu\, M_s^2}\gamma_{ik}\, M_l\, \sigma_{kl}^{in} + \frac{2\lambda_{\rm ex} M_s^2}{\mu\Omega}\gamma_{ik} M_l \sigma_{lk}^{ex}, \tag{24}$$

which after some algebra is converted into equation for $u$ having the form

$$\frac{du(t)}{dt} = -\left[\alpha\omega - \beta\frac{\omega^2}{\Omega}u(t)\right](u^2(t)-1). \tag{25}$$

The right hand side of this equation suggests that there are three stationary state characterized by

$$u_0(\theta=0) = \pm 1,\ \ u_0(\theta=\theta_M) = \frac{\alpha}{\beta}\frac{\Omega}{\omega}. \tag{26}$$

Making use of substitution $u(t) = u_0 + \delta u(t)$ one finds that linearized equation (25) can be identified as the equation of exponential relaxation, $\delta\dot{u} = -\tau^{-1}\delta u$, if and only if the parameter

$$\tau^{-1} = \left[2\left(\alpha\omega - \beta\frac{\omega^2}{\Omega}u_0\right)u_0 + \beta\frac{\omega^2}{\Omega}(1-u_0^2)\right] \tag{27}$$

is positive constant. It is easy to see that this is the case for $u_0(\theta = 0) = 1$ and $u_0(\theta = \theta_M)$ given by rightmost of equations (26). This latter $u_0$ corresponds to quasi-stationary transitional configuration of magnetization owing its existence to the coupling of magnetization with extrinsic stresses of magnetic anisotropy. For the total relaxation time $\tau^{-1} = \tau^{-1}(\theta = 0) + \tau^{-1}(\theta = \theta_M)$ and the FMR linewidth we obtain

$$\tau^{-1} = 2\omega\alpha - \beta\frac{\omega^2}{\Omega}(1+\cos^2\theta_M), \tag{28}$$

$$\begin{aligned}\Delta H &= \frac{2\omega}{\gamma\mu_0}\left(\alpha - \frac{\beta\omega}{2\Omega}\left[1+\cos^2\theta_M\right]\right) \\ &= \frac{4\pi f}{\gamma\mu_0}\left[\alpha - \frac{\beta}{2}\left(1-\left(\frac{\gamma\mu_0 M_s}{2\pi f}\right)^{1/2}\right)(1+\cos^2\theta_M)\right].\end{aligned} \tag{29}$$

In the reminder we focus on a case of in-plane configuration ($\mathbf{M}\uparrow\uparrow\mathbf{H}$) which is of particular interest in connection with recent discovery of non-linear frequency dependence of FMR linewidth[4,18]. In this case, the last equation for the FMR linewidth takes the form

$$\Delta H = \frac{4\pi f}{\gamma\mu_0}\left[\alpha - \beta(1-(\gamma\mu_0 M_s/2\pi f)^{1/2})\right]. \tag{30}$$

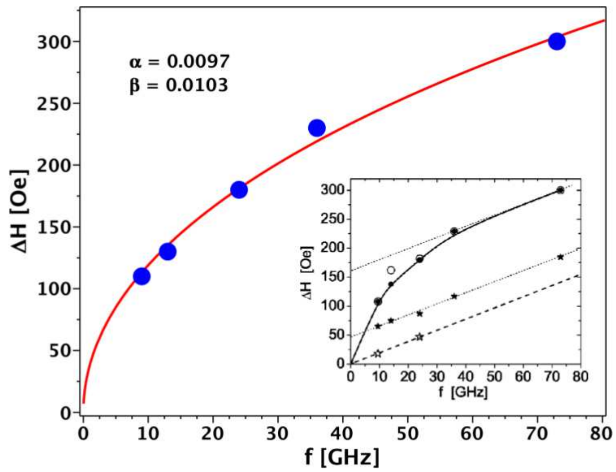


FIG. 2. Theoretical fit with the aid of equation (30) of empirical non-linear frequency dependence of FMR linewidth detected in the FMR measurements[18] on multilayered metallic nanostructures Pd/Fe/GaAs.

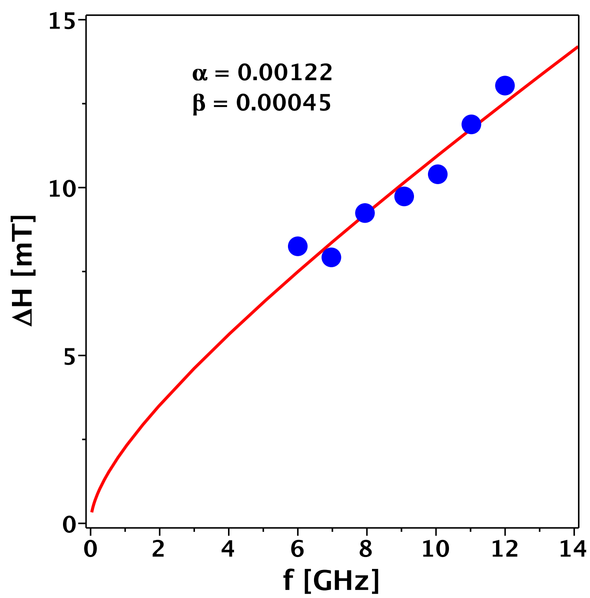


FIG. 3. The same as Fig.2, but for the thin films of ferromagnetic semiconductor (Ga,Mn)As investigated in[19].

## IV. DISCUSSION AND SUMMARY

To illustrate the difference between predictions of the standard and modified LL models, in Fig.1 we plot $\tau$ and $\Delta H$ as functions of FMR frequency computed with pointed out parameters of $\alpha$ and $\beta$. In computation on the basis of standard micromagnetic model, equation (22), we have used one and the same value of parameter $\alpha$ as in work[3] reporting the FMR measurements on ultrathin films of Permalloy. The pointed out in Fig.1 parameters $\alpha$ and $\beta$ have been deduced from fitting, with the aid of equation (30), of the non-linear frequency dependence of FMR linewidth discovered in FMR measurements[18] the results of which are pictured in Fig.2. In Fig.3 we plot theoretical fit of experimental data of FMR measurements on thin films of ferromagnetic semiconductor (Ga,Mn)As reported in[19]. All the above suggests that the developed micromagnetic theory of spin relaxation can be efficiently utilized as a guide in the experimental study of multilayered nanostructures by ferromagnetic resonance. A more detailed discussion of inferences following from this theory will be the subject of forthcoming article.

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