DAMPING OF ZERO SOUND IN A FERMI LIQUID

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The damping coefficient of zero sound in a Fermi liquid is obtained on the basis of Landau's phenomenological theory using perturbation theory. The expression obtained is exact, if corrections of order $(\omega\tau)^{-2}$ are not taken into account.

THE question of the absorption of zero sound in a Fermi liquid was first investigated in the classical work of Landau.[1] Subsequently, Abrikosov and Khalatnikov[2] found, on the basis of the Landau theory, the damping coefficient of zero-sound in the τ-approximation. Éliashberg[3] applied the microscopic Fermi-liquid theory to this problem. Theoretical interest in zero-sound has increased[4-6] since Abel, Anderson and Wheatley[7] detected zero sound in a Fermi liquid experimentally.

Brooker and Sykes[8] found exact values for the kinetic coefficients of a Fermi liquid in the limit of low temperatures, from which the exact value of the damping coefficient of hydrodynamic sound in the Fermi liquid is obtained directly. It is shown in the present paper that the damping coefficient of zero sound can be calculated to the same accuracy in perturbation theory with the small parameter equal to

$$\frac{m^{*3}T^2}{12\omega\pi^2\hbar^6}\int\frac{d\Omega}{2\pi}\frac{W(\theta,\varphi)}{\cos(\theta/2)}.$$

(Here m^* is the effective mass, T is the temperature, ω is the frequency, and θ is the angle between $\mathbf{p}$ and $\mathbf{p}_1$; φ is the angle between the planes $\mathbf{p}, \mathbf{p}_1$ and $\mathbf{p}', \mathbf{p}_1'$ and $W(\theta, \varphi)$ is the cross section for the scattering of quasiparticles at the Fermi surface.) If we go over into the τ-approximation, as in [2], this quantity will play the role of $1/\omega\tau$, and the general formula (6) for the damping coefficient of zero sound goes over, as it should, into the corresponding formula of Abrikosov and Khalatnikov.

We confine ourselves to calculating the principal term of the perturbation theory series and neglect corrections of relative order $(T/T_F)^2$ arising from the fact that the momenta of the interacting quasiparticles do not lie exactly on the Fermi surface.

We shall examine, for definiteness, the longitudinal sound. The kinetic equation for the Fourier component of the non-equilibrium correction to the distribution function

$$(\delta n)_{\mathbf{k},\omega}=\frac{\partial n_0}{\partial\varepsilon}\sum_{n=0}^{\infty}\nu_n(t)P_n(\cos\chi) \tag{1}$$

after simple transformations takes the form

$$(\xi-\cos\chi)\sum_{n=0}^{\infty}\nu_n(t)P_n(\cos\chi)-\cos\chi\left(F_0\langle\nu_0\rangle+\frac{F_1}{3}\langle\nu_1\rangle\cos\chi\right)$$
$$=\frac{m^{*3}T^2s}{8i\omega\pi^4\hbar^6}\int_{-\infty}^{\infty}dx\frac{(x-t)\operatorname{ch}(t/2)}{2\operatorname{sh}(x-t/2)\operatorname{ch}(x/2)}\sum_{n=0}^{\infty}\int\frac{d\Omega}{2\pi}\frac{W(\theta\ \varphi)}{\cos(\theta/2)}$$
$$\times\left[P_n(\cos\theta)-P_n\left(\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}\cos\varphi\right)-P_n\left(\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\cos\varphi\right)\right]\nu_n(x)P_n(\cos\chi), \tag{2}$$

where

$$F(\theta)=\frac{d\tau}{d\varepsilon}f(\theta)=\sum_n^{\infty}F_nP_n(\cos\theta),\qquad d\Omega=\sin\theta\,d\theta\,d\varphi,$$
$$n_0=(e^{(\varepsilon-\mu)/T}+1)^{-1},$$
$$\langle\nu_n\rangle=-\int_{-\infty}^{\infty}\frac{\partial n_0}{\partial t}\nu_n(t)\,dt,\qquad t=\frac{\varepsilon-\mu}{T},\qquad \cos\chi=\frac{(\mathbf{kv})}{kv},\qquad x=\frac{\varepsilon_1-\mu}{T},$$
$$\xi=s+\frac{ism^{*3}T^2}{16\omega\pi^2\hbar^6}\left(1+\frac{t^2}{\pi^2}\right)\int\frac{d\Omega}{2\pi}\frac{W(\theta\ \varphi)}{\cos(\theta/2)},\qquad s=\frac{\omega}{kv}.$$

In (2) we have put $F_n = 0$ for $n = 2, 3, \ldots$. We note that in principle, we can take into account any finite number of F_n.

We introduce the notation ($n = 0, 1, \ldots$):

$$\beta_n=\int\frac{d\Omega}{2\pi}\frac{W(\theta,\varphi)}{\cos(\theta/2)}\left[P_n(\cos\theta)-P_n\left(\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}\cos\varphi\right)-P_n\left(\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\cos\varphi\right)\right],$$
$$\frac{1}{\tau}=\frac{m^{*3}T^2}{12\pi^2\hbar^6}\int\frac{d\Omega}{2\pi}\frac{W(\theta,\varphi)}{\cos(\theta/2)}.$$

We divide (2) by $\xi - \cos\chi$ and average over $\cos\chi$ with weight $\frac{1}{2}P_n(\cos\chi)$. We have

$$\frac{\nu_n(t)}{2n+1}=\left(F_0\langle\nu_0\rangle+\frac{F_1}{3}\langle\nu_1\rangle s_0\right)s_0Q_n(s_0)+O\left(\frac{1}{\omega\tau}\right), \tag{3}$$

where s_0 is the velocity of sound at $T = 0$°K and $Q_n(s_0)$ is a Legendre function of the second kind (see, e.g., [9]).

In the kinetic equation we shall retain the terms linear in $1/\omega\tau$. Then, in the right-hand side of (2), we must substitute for $\nu_n(t)$ only the first terms of (3). After the substitution, (2) acquires the form

$$(\xi-\cos\chi)\sum_{n=0}^{\infty}\nu_n(t)P_n(\cos\chi)-\cos\chi\left(F_0\langle\nu_0\rangle+\frac{F_1}{3}\langle\nu_1\rangle\cos\chi\right)$$
$$=\frac{3is_0}{4\omega\tau}\left(1+\frac{t^2}{\pi^2}\right)\left[\langle\nu_0\rangle+\frac{3s_0}{1+F_1/3}\langle\nu_0\rangle\cos\chi+\sum_{n=2}^{\infty}\frac{\beta_n(2n+1)}{\beta_0w(s_0)}\right.$$
$$\left.\times s_0Q_n(s_0)\langle\nu_0\rangle P_n(\cos\chi)\right]. \tag{4}$$

We multiply (4) successively by $(\xi - \cos\chi)^{-1}$ and $\cos\chi(\xi - \cos\chi)^{-1}$ and average over $\cos\chi$. We linearize the resulting equations with respect to $1/\omega\tau$ and integrate over t with weight $\partial n_0/\partial t$; finally, from the condition that the equations be consistent, we obtain the dispersion equation

$$(F_0 w-1)\left(1+\frac{F_1}{3}\right)+F_1 s_0^2 w+\left[\frac{1}{w}\left(\frac{w+1}{s_0}-\frac{s_0}{s_0^2-1}\right)+\frac{2s_0 w F_1}{1+F_1/3}\right]$$
$$\times\left(1+\frac{F_1}{3}\right)is'+\frac{is_0}{\omega\tau}\left[\frac{1}{w}\left(\frac{w+1}{s_0}-\frac{s_0}{s_0^2-1}\right)\right.$$
$$\left.+\sum_{n=0}^{\infty}\frac{\beta_n(2n+1)}{\beta_0 w}s_0 Q_n^2(s_0)\right]=0,$$

$$is'=s-s_0,\quad |s'|\ll|s_0|,\quad w=-1+\frac{s_0}{2}\ln\frac{s_0+1}{s_0-1}. \tag{5}$$

From (5) it is clear that the equation for the velocity of sound has remained unchanged compared with [2], and for the damping coefficient $\alpha = \mathrm{Im}\, k$ we have

$$\alpha=\frac{s_0 m^{*3}T^2}{12\pi^2\hbar^6 v_F}\int\frac{d\Omega}{2\pi}\frac{W(\theta,\varphi)}{\cos(\theta/2)}\left[\frac{1}{s_0^2-1}-\sum_{n=0}^{\infty}\frac{\beta_n}{\beta_0}(2n+1)Q_n^2(s_0)\right]$$
$$\times\left(\frac{1}{s_0^2-1}-w-\frac{2s_0^2w^2F_1}{1+F_1/3}\right)^{-1}. \tag{6}$$

By putting $\beta_n = 0$, where $n = 2, 3, \ldots$, in (6), we arrive at a formula which coincides with the corresponding formula in the paper by Abrikosov and Khalatnikov.

The true dependence of the cross section $W(\theta, \varphi)$ on the angles is not known. To compare the result obtained with experiment, we take the cross section $W(\theta, \varphi)$ in its simplest form. We assume, in the first place, that the cross section does not depend on the azimuthal angle φ. According to general scattering theory (cf., e.g., [10]), the total scattering cross section for unpolarized spin-½ particles is

$$W(\theta)=\frac{2\pi}{\hbar}\left(\frac{3}{4}|A_{\uparrow\uparrow}(\theta)|^2+\frac{1}{4}|A_{\uparrow\downarrow}(\theta)|^2\right), \tag{7}$$

where $A_{\uparrow\uparrow}(\theta)$ and $A_{\uparrow\downarrow}(\theta)$ are the scattering amplitudes with parallel and antiparallel spins respectively. We represent them in the form of an expansion in Legendre polynomials, following Abrikosov and Khalatnikov

$$A_{\uparrow\uparrow,\uparrow\downarrow}(\theta)=\frac{\pi^2\hbar^3}{m^* p_F}\sum_{n=0}^{\infty}\left(\frac{F_n}{1+F_n/(2n+1)}\pm\frac{Z_n}{4+Z_n/(2n+1)}\right)P_n(\cos\theta)$$

We make use of recent experimental values [11] of F_0, F_1, Z_0, and Z_1 (Z_1 is determined from the condition $A_{\uparrow\uparrow}(0) = 0$) and, as is usual, put all the succeeding terms of the sum equal to zero. As a result, we find for He^3 at a pressure of 0.28 atm

$$W(\theta)=\frac{\pi^5\hbar^5}{m^{*2}p_F^2}(5.92+5.24\cos\theta+6.19\cos^2\theta). \tag{8}$$

We note that, in all the papers on Fermi liquids known to us in which $W(\theta, \varphi)$ is calculated (including also a paper of one of the authors[5]), the scattering amplitudes occur in (7) with other weights, and this leads to different numerical coefficients in (8).[1)]

Analysis of the sum in (6) shows that for $s_0^2 \gg 1$, to calculate α accurate to within 10% it is sufficient to take the first three terms of the sum. Substituting the cross section (8) and the values[11] $v_F = 5.38 \times 10^3$ cm/sec and $p_F/\hbar = 7.88 \times 10^7\ \mathrm{cm}^{-1}$ in (6), we find $\alpha T^{-2} = 0.8 \times 10^6\ \mathrm{cm}^{-1}\,\mathrm{deg}^{-2}$; the experimental value of αT^{-2} is[11] $1.5 \times 10^6\ \mathrm{cm}^{-1}\,\mathrm{deg}^{-2}$.

The discrepancy between the experimental and theoretical values of α can be due to two reasons: firstly, using the cross section in the crudely approximate form (8), we should expect agreement with experiment only in order of magnitude; secondly, the experimental accuracy[7, 12] is not great, since the error in the determination of the temperature, according to Wheatley,[12] is of the same order as the temperatures themselves at which damping of the zero sound was measured.

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1)Substituting the cross section (8) in the formulas of Brooker and Sykes [8] for the viscosity η and thermal conductivity κ we find $\eta T^2 = 1.6 \times 10^{-6}$ poise-deg² and $T\kappa = 38$ g-cm-sec⁻³, which agrees better with the experimental values [12] $\eta T^2 = 2.0 \times 10^{-6}$ poise-deg², $\kappa T = 35$ g-cm-sec⁻³, than do the values $\eta T^2 = 2.5 \times 10^{-6}$ poise-deg², $\kappa T = 50$ g-cm-sec⁻³ calculated in [8].

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